**6.3. For the production function *Q* = 6*L*2 − *L*3, fill in the following table and state how much the firm should produce so that:**

**a) average product is maximized**

**b) marginal product is maximized**

**c) total product is maximized**

**d) average product is zero**

****

The completed table is shown below:

|  |  |
| --- | --- |
| L  0  1  2  3  4  5  6 | Q  0  5  16  27  32  25  0 |

a) You can calculate the average product at each point by just dividing total output by L. The values obtained are 0,5,8,9,8,5,0. Therefore Average Product is maximized when L = 3.

b) The marginal product at values 1 through 6 are respectively: 5,11,11,5,–7, –25. Therefore both the second and the third unit of L give the greatest marginal increase in output [if you use calculus techniques it can be seen that marginal product is maximized when *L* = 2].

c) From the Table it is clear that total product is maximized when L = 4.

d) Average Product will be zero only when Total Product is zero. This happens when L = 6.

**6.7. The following table shows selected input quantities, total products, average products, and marginal products. Fill in as much of the table as you can:**

****

****

The correct answers are shown in bold face red type.

|  |  |  |  |
| --- | --- | --- | --- |
| Labor, *L* | Total product, *Q* | *APL* | *MPL* |
| 0 | 0 | 0 | ---- |
| 1 | 19 | **19** | 19 |
| 2 | **72** | 36 | **53** |
| 3 | **153** | **51** | **81** |
| 4 | 256 | 64 | 103 |
| 5 | 375 | **75** | **119** |
| 6 | **504** | **84** | 129 |
| 7 | 637 | 91 | 133 |
| 8 | **768** | 96 | **131** |
| 9 | 891 | **99** | **123** |
| 10 | **1000** | 100 | **109** |
| 11 | 1089 | **98** | 89 |
| 12 | **1152** | 96 | **63** |
| 13 | **1183** | **91** | **31** |
| 14 | **1176** | **84** | -7 |
| 15 | **1125** | 75 | -51 |

**6.14** **Consider the following production functions and their associated marginal products.**

**For each production function, determine the marginal rate of technical substitution of labor for capital, and indicate whether the isoquants for the production function exhibit diminishing marginal rate of substitution.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Production function | *MPL* | *MPK* | *MRTSL,K* | Diminishing marginal product of labor? | Diminishing marginal product of capital? | Diminishing marginal rate of technical substitution |
|  |  |  |  | NO | NO | NO |
|  |  |  |  | YES | YES | YES |
|  |  |  |  | YES | YES | YES |
|  |  |  |  | NO | NO | YES |
|  |  |  |  | NO | NO | NO |

**6.23. A firm’s production function is *Q* = 5*L*2*/*3*K*1*/*3 with *MPK* = (5*/*3)*L*2*/*3*K*−2*/*3 and *MPL* = (10*/*3)*L*−1*/*3*K*1*/*3.**

**a) Does this production function exhibit constant, increasing, or decreasing returns to scale?**

**b) What is the marginal rate of technical substitution of *L* for *K* for this production function?**

**c) What is the elasticity of substitution for this production function?**

a) Notice that (*aK*)1/3(*aL*)2/3 = *a*1/3*a*2/3 *K*1/3 *L*2/3 = *a* *K*1/3 *L*2/3 = *aQ*. This production function exhibits constant returns to scale.

b) *MRTSL,K* = *MPL* / *MPK* = 2 *K*/*L*.

1. Because this is a Cobb-Douglas production function, its elasticity of substitution equals 1.

**6.27. Suppose a firm’s production function initially took the form *Q* = 500(*L* + 3*K*). However, as a result of a manufacturing innovation, its production function is now *Q* = 1000(0*.*5*L* + 10*K*).**

**a) Show that the innovation has resulted in technological progress in the sense defined in the text.**

**b) Is the technological progress neutral, labor saving, or capital saving?**

a) It is possible to write the two production functions as



Since  for given quantities of  and , the firm can achieve more output for a given combination of inputs. This innovation has therefore resulted in technological progress as defined in the text.

b) Initially  and  implying the . After the innovation the  and  implying the . Since the marginal rate of technical substitution of labor for capital has decreased after the innovation this is labor-saving technological progress.

**7.5. A firm uses two inputs, capital and labor, to produce output. Its production function exhibits a diminishing marginal rate of technical substitution.**

**a) If the price of capital and labor services both increase by the same percentage amount (e.g., 20 percent), what will happen to the cost-minimizing input quantities for a given output level?**

**b) If the price of capital increases by 20 percent while the price of labor increases by 10 percent, what will happen to the cost-minimizing input quantities for a given output level?**

a) If the price of both inputs change by the same percentage amount, the slope of the isocost line will not change. Since we are holding the level of output fixed, the isocost line will be tangent to the isoquant at the same point as prior to the price increase. Therefore, the cost-minimizing quantities of the inputs will not change.

b) If the price of capital increases by a larger percentage than the price of labor, then, relatively speaking, the price of labor has become cheaper. The firm will substitute away from capital and add labor until either the tangency condition holds or a corner solution is reached.

**7.12. A firm operates with the production function Q = K2L. Q is the number of units of output per day when the firm rents K units of capital and employs L workers each day. The marginal product of capital is 2KL, and the marginal product of labor is K2. The manager has been given a production target: Produce 8,000 units per day. She knows that the daily rental price of capital is $400 per unit. The wage rate paid to each worker is $200 day.**

**a) Currently the firm employs at 80 workers per day. What is the firm’s daily total cost if it rents just enough capital to produce at its target?**

**b) Compare the marginal product per dollar sent on K and on L when the firm operates at the input choice in part (a). What does this suggest about the way the firm might change its choice of K and L if it wants to reduce the total cost in meeting its target?**

**c) In the long run, how much K and L should the firm choose if it wants to minimize the cost of producing 8,000 units of output day? What will the total daily cost of production be?**

a) Suppose that the firm is operating in the short run, with L = 80. To produce Q = 8000, how much K will it require? From the production function we observe that 8,000 = K2 (80) => K = 10.

The total cost would be C = wL + rK = $200(80) + $400(10) = $2,000 per day.

b) Let’s examine the “bang for the buck” for K and L when K = 10 and L = 80.

For capital: MPK / r = 2KL / 400 = 2(10)(80) / 400 = 4

For labor: MP­L / w = K2 / 200 = 102 / 200 = 0.5

So the marginal product per dollar spent on capital exceeds that of labor. The firm would like to rent more capital and hire fewer workers.

c) Because the production function is Cobb-Douglas, we know that it has diminishing MRTSL,K and that the isoquants do not intersect either the K or L axis. Thus the cost reducing basket (K,L) will be interior (with K > 0 and L > 0). To find the optimum, we use the two conditions:

(1) Tangency condition: MPK / MP­L = r / w => 2KL/K­­­2 = 400 / 200 => K = L

(2) Production Requirement: K2L = 8,000

Together equations (1) and (2) tell us that K = 20 and L = 20.

The total cost would be C = wL + rK = $200(20) + $400(20) = $12,000 per day.

**7.16. A construction company has two types of employees: skilled and unskilled. A skilled employee can build 1 yard of a brick wall in one hour. An unskilled employee needs twice as much time to build the same wall. The hourly wage of a skilled employee is $15. The hourly wage of an unskilled employee is $8.**

**a) Write down a production function with labor. The inputs are the number of hours of skilled workers, *LS*, the number of hours worked by unskilled employees, *LU*, and the output is the number of yards of brick wall, *Q*.**

**b) The firm needs to build 100 yards of a wall. Sketch the isoquant that shows all combinations of skilled and unskilled labor that result in building 100 yards of the wall.**

**c) What is the cost-minimizing way to build 100 yards of a wall? Illustrate your answer on the graph in part (b).**

a) The production function is *Q* = *LS* + ½ *LU* where *LS* denotes hours worked by skilled workers and *LU* denotes hours worked by unskilled workers. Both types of labor are perfect substitutes.

b) The isoquant is a straight line.

*LU*

*LS*

100h

200h

Isoquant representing 100 yards of wall

Isoccost line representing $1500 expenditure

175h

c) *MPLs/ws = 1/15*; *MPLu/wu = 0.5/8 = 1/16*. Thus, the “bang for the buck” is higher for skiled labor, and the firm will use only skilled labor.   
Note that the total cost of building 100 yards with skilled labor is (100 hours)($15/ hour) = $1500.   
The total cost of building 100 yards with unskilled labor is (200 hours)($8/ hour) = $1600.  
  
The isocost line representing a $1500 expenditure is drawn as a dotted line in the graph in (b). The isocost line is more steeply sloped than the isoquant in the graph because the marginal rate of technical substitution of unskilled labor for unskilled labor is equal to ½, while the ratio of input prices is equal to 8/15.

**7.24. Consider the production function *Q* = *K* + √*L*. For this production function, *MPL* = 1*/*(2√*L*) and *MPK* = 1. Derive the input demand curves for *L* and *K*, as a function of the input prices *w* (price of labor services) and *r* (price of capital services). Show that at an interior optimum (with *K >* 0 and *L >* 0) the amount of *L* demanded does not depend on *Q*. What does this imply about the expansion path?**

The tangency condition implies that . Clearly the demand curve for *L* is not a function of the level of output, *Q*. Therefore, as the level of output changes, the amount of labor is constant. Therefore, if we were to graph isoquants with labor on the horizontal axis, the expansion path for labor would just be a straight, vertical line.

The demand curve for capital can be derived by substituting the demand curve for labor into the production function. That is, .

**7.28. A plant’s production function is *Q* = 2*KL* + *K* . For this production function, *MPK* = 2*L* + 1 and *MPL* = 2*K*. The price of labor services *w* is $4 and of capital services *r* is $5 per unit.**

**a) In the short run, the plant’s capital is fixed at *K* = 9. Find the amount of labor it must employ to produce *Q* = 45 units of output.**

**b) How much money is the firm sacrificing by not having the ability to choose its level of capital optimally?**

a) Since , we get which implies that *L* = 36/18 = 2. Therefore the firm’s total cost with this input combination is 4(2) + 5(9) = $53.

b) If the firm could operate optimally, it would choose labor and capital to satisfy the tangency condition: , implying that  Also,  Combining these two conditions, **= 4.24 and *L* = 4.8. Now the firm’s expenditure would be 4(4.24) + 5(4.8) = $41 approximately. Therefore the firm loses about $12 because of its constraint on capital.

**8.10. For each of the total cost functions, write the expressions for the total fixed cost, average variable cost, and marginal cost (if not given), and draw the average total cost and marginal cost curves.**

**a) *TC*(*Q*) = 10*Q***

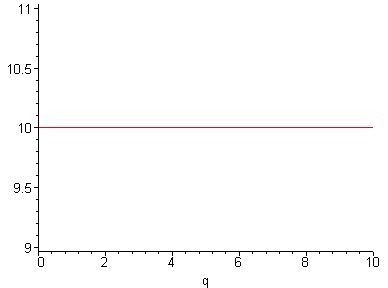
**b) *TC*(*Q*) = 160 + 10*Q***

**c) *TC*(*Q*) = 10*Q*2, where *MC*(*Q*) = 20*Q***

**d) *TC*(*Q*) = 10√*Q*, where *MC*(*Q*) = 5*/*√*Q***

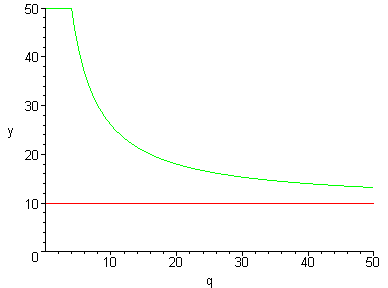
**e) *TC*(*Q*) = 160 + 10*Q*2, where *MC*(*Q*) = 20*Q***

a) *TFC =* 0, *AVC =* 10, *MC =* 10.



*MC = AC* = 10

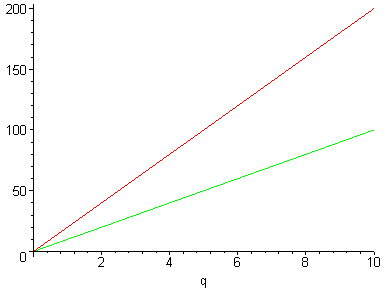
b) *TFC =* 160, *AVC =* 10, *MC =* 10.



*AC*

*MC* = 10

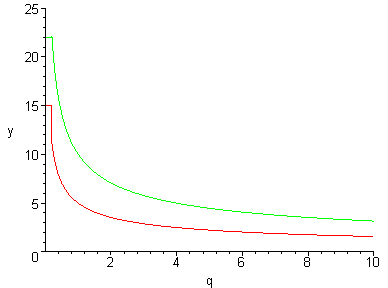
c) *TFC =* 0, *AVC =* 10*Q*.



*AC*

*MC*

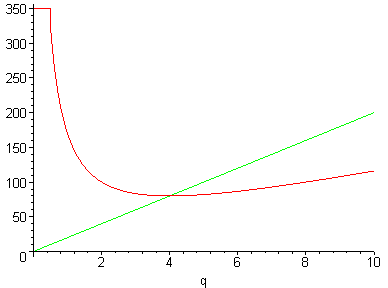
d) *TFC =* 0, *AVC =* .



*AC*

*MC*

e) *TFC =* 160, *AVC =* 10*Q*.



*MC*

*AC*

**8.16. A hat manufacturing firm has the following production function with capital and labor being the inputs: *Q* = min(4*L*, 7*K* )—that is it has a fixed-proportions production function. If *w* is the cost of a unit of labor and *r* is the cost of a unit of capital, derive the firm’s long-run total cost curve and average cost curve in terms of the input prices and *Q*.**

The fixed proportions production function implies that for the firm to be at a cost minimizing optimum, and both of these equal *Q*. Therefore, *L* = *Q*/4 and *K* = *Q*/7. So the firm’s total cost is .

The average cost curve is  Note that this average cost curve is independent of Q and is simply a straight line.

**8.17. A packaging firm relies on the production function *Q* = *KL* + *K*, with *MPL* = *K* and *MPK* = *L* + 1. Assume that the firm’s optimal input combination is interior (it uses positive amounts of both inputs). Derive its long-run total cost curve in terms of the input prices, *w* and *r*. Verify that if the input prices double, then total cost doubles as well.**

Since we can assume an interior solution, the tangency condition must hold. Therefore the optimal bundle must be such that  This means  Substituting this back into the production function, we see that .

This implies that The total cost curve is then  =  If we substitute 2*w* and 2*r* in the place of *w* and *r* respectively, we get *TC*2 = , so total cost does indeed double when input prices double.

**8.21. When a firm uses *K* units of capital and *L* units of labor, it can produce *Q* units of output with the production function *Q* = √*L* + √*K* . Each unit of capital costs 2, and each unit of labor costs 1.**

**a) The level of *K* is fixed at 16 units. Suppose *Q* ≤ 4. What will the firm’s short-run total cost be? (Hint: How much labor will the firm need?)**

**b) The level of *K* is fixed at 16 units. Suppose *Q >* 4. Find the equation of the firm’s short-run total cost curve.**

a) Even if the firm hires zero units of labor, with *K* fixed at 16 it can still produce up to *Q = *= 4 units of output. So for , *L =* 0 is the cost-minimizing choice of labor and the short-run total cost function is just the cost of capital: *C = rK + wL =* 2(16) + 1(0) = 32.

b) For *Q >* 4, the firm needs to hire positive amounts of labor, according to  or *L =* (*Q –* 4)2. So for *Q >* 4, the short-run total cost function is *C*(*Q*) = *rK + wL =* 2(16) + 1(*Q –* 4)2 = 32 + (*Q –* 4)2.

**9.6. A bicycle-repair shop charges the competitive market price of $10 per bike repaired. The firm’s short-run total cost is given by *STC*(*Q*) = *Q*2*/*2, and the associated marginal cost curve is *SMC*(*Q*) = *Q*.**

**a) What quantity should the firm produce if it wants to maximize its profit?**

**b) Draw the shop’s total revenue and total cost curves, and graph the total profit function on the same diagram. Using your graph, state (approximately) the profit-maximizing quantity in each case.**

a) Since the firm is producing in a perfectly competitive market, the firm views the output price as exogenous. It should produce up to the point at which *P = SMC(Q)*, that is, so that 10 = *Q*. So it should produce 10 units of output.

b) The graph is shown below.



The total cost function increases in Q, and at an increasing rate. Total Profit at first increases in Q and then decreases. From the graph, it appears that Profit is maximized when Q is about 10, which we found in (a).

**9.16. The wood-pallet market contains many identical firms, each with the short-run total cost function *STC*(*Q*) = 400 + 5*Q* + *Q*2, where *Q* is the firm’s annual output (and all of the firm’s $400 fixed cost is sunk). The corresponding marginal cost function is *SMC*(*Q*) = 5 + 2*Q*. The market demand curve for this industry is *D*(*P*) = 262*.*5 − *P/*2, where *P* is the market price. Each firm in the industry is currently earning zero economic profit. How many firms are in this industry, and what is the market equilibrium price?**

Since each firm is earning zero economic profit, we know . Since each firm supplies where , set .



Since , . If market price is , . Finally, if total market demand is  and each firm is producing 20 units, there will be  firms in the market.

**9.19. A competitive industry consists of 6 type A firms and 4 type B firms. Each firm of type A operates with the supply curve:**

****

**Each firm of type B operates with the supply curve:**

****

**a) Suppose the market demand is **

**At the market equilibrium, which firms are producing, and what is the equilibrium price?**

**b) Suppose the market demand is **

**At the market equilibrium, which firms are producing, and what is the equilibrium price?**

a) When *P* <10, only Type B firms will operate, and the market supply will be 4(2*P*) = 8*P*.  
When *P* >10, both types of firms will operate, and the market supply will be   
4(2*P*) + 6(-10 + *P*) = -60 + 14*P*.  
To summarize, the market supply will be   
Let’s first assume the equilibrium price exceeds 10, so that all firms are producing. If this is true, setting market supply equal to market demand: -60 + 14*P* = 108 – 10*P*, so that *P* = 7; however, the market supply we have used is valid for P>10, but not valid for *P* = 7.  
So the equilibrium price must be less than 10, with only Type B firms producing (and Type A firms not producing).  
Setting market supply equal to market demand: 8*P* = 108 – 10*P*, so that *P* = 6.  
We have found that in equilibrium, only Type B firms produce, and the equilibrium price is 6.

b) Let’s first assume the equilibrium price exceeds 10, so that all firms are producing. If this is true, setting market supply equal to market demand: -60 + 14*P* = 228 – 10*P*, so that *P* = 12; the market supply we have used is valid for P=12. At this equilibrium both types of firms will be producing.

**9.32. The long-run average cost for production of hard-disk drives is given by *AC*(*Q*) = √*wr*(120−20*Q* + *Q*2), where *Q* is the annual output of a firm, *w* is the wage rate for skilled assembly labor, and *r* is the price of capital services. The corresponding long-run marginal cost curve is *MC*(*Q*) = √*wr*(120 − 40*Q* + 3*Q*2). The demand for labor for an individual firm is**

****

**The price of capital services is fixed at *r* = 1.**

**a) In a long-run competitive equilibrium, how much output will each firm produce?**

**b) In a long-run competitive equilibrium, what will be the market price? Note that your answer will be expressed as a function of *w*.**

**c) In a long-run competitive equilibrium, how much skilled labor will each firm demand? Again, your answer will be in terms of *w*.**

**d) Suppose that the market demand curve is given by *D*(*P*) = 10,000*/P*. What is the market equilibrium quantity as a function of *w*?**

**e) What is the long-run equilibrium number of firms as a function of *w*?**

**f ) Using your answers to parts (c) and (e), determine the overall demand for skilled labor in this industry as a function of *w*.**

**g) Suppose that the supply curve for the skilled labor used in this industry is *Γ*(*w*) = 50*w*. At what value of *w* does the supply of skilled labor equal the demand for skilled labor?**

**h) Using your answer from part (g), go back through parts (b), (d), and (e) to determine the long-run equilibrium price, market demand, and number of firms in this industry.**

**i) Repeat the analysis in this problem, now assuming that the market demand curve is given by *D*(*P*) = 20,000*/P*.**

a) In a long-run competitive equilibrium  and , implying .



b) In a long-run competitive equilibrium  so that (with  and )



c) Given demand for labor and setting  and 



d) Given market demand and setting 



e) Since each firm will produce 10 units,



f) From part c), the labor demand for an individual firm is . Overall demand for labor is then



g) Setting the supply of skilled labor equal to the demand for skilled labor,



h) Plugging  into the solution for price implies ; plugging  in market demand implies ; and plugging  into the solution for the number of firms and rounding down to the nearest integer implies .

i) If



The number of firms will be



Overall labor demand will be



Setting the supply of labor equal to the demand for labor implies



Plugging  into the solution for price implies ; plugging  into market demand implies ; and plugging  into the solution for the number of firms and rounding down to the nearest integer implies .